

New Third-Order Moments for the CBL

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Abstract

Turbulent convection is an inherently non-local phenomenon and a primary condition for a successful treatment of the CBL (convective boundary layer) is a reliable model of non-locality. In the dynamic equations governing the convective flux, the turbulent kinetic energy, etc., non-locality is represented by the third-order moments, TOMs. Since the simplest form, the so-called down gradient approximation (DGA), severely underestimates the TOMs (up to an order of magnitude), a more physical model is needed. In 1994, an analytical model was presented which was derived directly from the dynamical equations for the TOMs. It considerably improved the DGA but was a bit cumbersome to use and, more importantly, it was based on the quasi-normal (QN) approximation for the fourth-order moments.

Here, we present a new analytic expression for the TOMs which is structurally simpler than the 1994 expression and which avoids the QN approximation. The resulting fit to the LES data is superior to that of the 1994 model.

I. Introduction.

The search for a reliable expression for the third-order moments to be used in the dynamic equations for the second-order moments such as the turbulent kinetic energy, the convective fluxes, etc., has a long history. For many years, people used the so-called down gradient approximation but LES (large eddy simulations) have shown that said model severely underestimates the TOMs (Moeng and Wyngaard, 1989).

Prompted by these results, Canuto et al.(1994) undertook the task of solving directly the dynamic equations for the TOMs thus avoiding the need for phenomenological expressions. The key merit of the 1994 model was to exhibit the fact that all the TOMs are a linear combination of the gradients of all the second-order moments; and not only of selected ones, as assumed in the down-gradient approximation. From the performance viewpoint, the new TOMs reproduced the LES data quite satisfactorily but the predicted $\overline{w^2\theta}$ and $\overline{w\theta^2}$ were not as good as that of the other TOMs. The weakest point in the 1994 model was the use of the QN (quasi-normal) approximation for the fourth-order moments. Both these limitations motivated us to search for new expressions for the TOMs which are simpler and with a better physical content.

II. The new physical ingredient of the third-order moments

There are six TOMs to be considered:

$$\overline{w^3}, \overline{q^2w}, \overline{w^2\theta}, \overline{w\theta^2}, \overline{\theta^3}, \overline{q^2\theta} \quad (1)$$

Here, u, v, w and θ are the fluctuating velocity and temperature fields and $q^2 = u^2 + v^2 + w^2$. Since the TOMs in (1) have different dimensions, we multiply the last four by appropriate variables so that all the TOMs have dimensions of a velocity cubed. Thus, we introduce the new variables x 's which have the same dimensions:

$$x_1 \equiv g\alpha\tau_v \overline{w^2\theta}, \quad x_2 \equiv (g\alpha\tau_v)^2 \overline{w\theta^2} \quad (2a)$$

$$x_3 \equiv (g\alpha\tau_v)^3 \overline{\theta^3}, \quad x_4 \equiv g\alpha\tau_v \overline{q^2\theta} \quad (2b)$$

$$x_5 \equiv \overline{wq^2}, \quad z \equiv \overline{w^3} \quad (2c)$$

Here, α is the volume expansion coefficient, g is the local gravity and τ_v a time scale that will be discussed below. The original dynamic equations for the TOM given in (1) can be found in Canuto (1992), Eqs.(37a),(38a),(39a) and (40a). These equations entail fourth order moments which can be written in general as:

$$\overline{abcd} = (\overline{ab} \overline{cd} + \overline{ac} \overline{bd} + \overline{ad} \overline{bc})f \quad (3a)$$

When the function f is taken to be unity, Eq.(3a) corresponds to the quasi-normal (QN) approximation. Under the QN approximation, the time scale τ_v introduced before can only be identified with the dynamic time scale of turbulence $\tau=2K/\epsilon$ (K and ϵ are the turbulent kinetic energy and its rate of dissipation).

With $\tau_v=\tau$, the explicit form of the TOMs was presented in Canuto et al.(1994). Even though the comparison with LES data was overall satisfactory, the predicted $\overline{w^2}$ and $\overline{w\theta^2}$ (figs.10–11 of Canuto et al., 1994) were not as good as that of the other TOMs. In addition, the expressions of the TOMs were rather cumbersome to use due to the fully explicit form of the determinant obtained when solving the algebraic set of equations for the TOMs.

For these reasons, we felt motivated to search for new expressions for the TOMs that are both simpler to handle and possess a better physical content. Zilitinkevich et al. (1999) tried to do so but their solution is not practical since it does not provide an expression for $\overline{w^3}$ which must be derived from outside, say from an LES.

To improve the 1994 model, we were guided by the fact that with $f=1$, the TOM can become arbitrarily large while in reality they have finite values, e.g., the small value of the skewness. Thus, reducing the TOM given by the $f=1$ case is a way to avoid unphysical results. How to directly relate this "damping effect" to a $f \neq 1$ is not a matter that can be carried out analytically, rather, we have used f in (3a) to formally indicate the physical motivation of our new approach. In the most successful heuristic model used to cut down the growth of the TOM, the EDQNM model (Lesieur, 1992), the damping is represented by an additional time scale which one must choose on physical grounds. Here, we suggest to

take:

$$\tau_v = \tau[1 + \lambda_0 N^2 \tau^2]^{-1}, \quad N^2 = -g\alpha \partial T / \partial z, \quad \tau = 2K/\epsilon \quad (3b)$$

We shall further take:

$$\lambda_0 = 0.04 \text{ if } N^2 > 0, \quad \lambda_0 = 0 \text{ if } N^2 < 0 \quad (3c)$$

The previous model (Canuto et al., 1994) corresponds to $f=1$ and/or to $\lambda_0=0$.

III. The new expression for the third-order moments

The new analytic expressions for the TOMs are quite simple:

$$\begin{aligned} x_1 &= X_0 z - X_1 \\ x_2 &= Y_0 z - Y_1 \end{aligned} \quad (4a)$$

$$\begin{aligned} x_3 &= Z_0 x_2 - Z_1 \\ x_4 &= W_0 x_5 + c^{-1} x_2 + W_1 \end{aligned} \quad (4b)$$

$$\begin{aligned} x_5 &= \Omega_0 z - \Omega_1 \\ z &= (\Omega_1 - 1.2X_1 - \frac{3}{2}f_5)(c - 1.2X_0 + \Omega_0)^{-1} \end{aligned} \quad (4c)$$

where the functions X , Y , Z , W and Ω are defined as:

$$\begin{aligned} X_0 &= \gamma_2 \tilde{N}^2 (1 - \gamma_3 \tilde{N}^2) [1 - (\gamma_1 + \gamma_3) \tilde{N}^2]^{-1} \\ X_1 &= [\gamma_0 f_0 + \gamma_1 f_1 + \gamma_2 (1 - \gamma_3 \tilde{N}^2) f_2] [1 - (\gamma_1 + \gamma_3) \tilde{N}^2]^{-1} \end{aligned} \quad (5a)$$

$$\begin{aligned} Y_0 &= 2\gamma_2 \tilde{N}^2 (1 - \gamma_3 \tilde{N}^2)^{-1} X_0 \\ Y_1 &= 2\gamma_2 (1 - \gamma_3 \tilde{N}^2)^{-1} (\tilde{N}^2 X_1 + \gamma_0 \gamma_1 f_0 + f_1) \end{aligned} \quad (5b)$$

$$\begin{aligned} Z_0 &= \frac{3}{2} (c - 2)^{-1} \tilde{N}^2 \\ Z_1 &= \frac{3}{2} (c - 2)^{-1} f_0 \end{aligned} \quad (5c)$$

$$\begin{aligned} \Omega_0 &= \omega_0 X_0 + \omega_1 Y_0 \\ \Omega_1 &= \omega_0 X_1 + \omega_1 Y_1 + \omega_2 \end{aligned} \quad (5d)$$

$$W_0 = \frac{1}{2c} \tilde{N}^2, \quad W_1 = -c^{-1} f_3 \quad (5e)$$

The auxiliary functions ω 's are:

$$\omega_0 = \gamma_4 (1 - \gamma_5 \tilde{N}^2)^{-1}, \quad \omega_1 = (2c)^{-1} \omega_0, \quad \omega_2 = \omega_1 f_3 + \frac{5}{4} \omega_0 f_4 \quad (6)$$

Finally, the γ 's are constants which depend on the only adjustable parameter c :

$$\begin{aligned}
\gamma_0 &= 0.52c^{-2}(c-2)^{-1}, & \gamma_1 &= 0.87c^{-2} & \gamma_2 &= 0.5c^{-1} \\
\gamma_3 &= 0.60c^{-1}(c-2)^{-1} \\
\gamma_4 &= 2.4(3c+5)^{-1}, & \gamma_5 &= 0.6c^{-1}(3c+5)^{-1}
\end{aligned} \tag{7}$$

Based on previous work, the suggested value is $c=7$ but small variations are allowed. The second order moments enter through the functions $f_{0,\dots,5}$ which are defined as follows:

$$\begin{aligned}
f_0 &= (g\alpha)^3 \tau_v^4 J \frac{\partial \bar{\theta}^2}{\partial z} \\
f_1 &= (g\alpha)^2 \tau_v^3 \left(J \frac{\partial J}{\partial z} + \frac{1}{2} \bar{w}^2 \frac{\partial \bar{\theta}^2}{\partial z} \right) \\
f_2 &= g\alpha \tau_v^2 J \frac{\partial \bar{w}^2}{\partial z} + 2g\alpha \tau_v^2 \bar{w}^2 \frac{\partial J}{\partial z} \\
f_3 &= g\alpha \tau_v^2 \left(\bar{w}^2 \frac{\partial J}{\partial z} + J \frac{\partial K}{\partial z} \right) \\
f_4 &= \tau_v \bar{w}^2 \left(\frac{\partial \bar{w}^2}{\partial z} + \frac{\partial K}{\partial z} \right) \\
f_5 &= \tau_v \bar{w}^2 \frac{\partial \bar{w}^2}{\partial z}
\end{aligned} \tag{8}$$

All the functions f 's have dimensions of velocity cubed. Finally:

$$J = \bar{w}\bar{\theta}, \quad \bar{N}^2 \equiv \tau_v^2 N^2 \tag{9}$$

IV. Test of the new TOM vs LES data.

In Fig.1 we compare the new TOMs given by Eqs.(2) and (4) vs. LES data. Rather than solving the CBL dynamic equations as it was done in the 1994 paper, here we employed LES data to compute the second-order moments Eq.(8), $\tau=2K/\epsilon$ and $N^2(z)$.

The better agreement with LES data with respect to the 1994 model, especially in Figs 1c–d, is also partly due to the simpler analytical form of the TOM which has allowed us to test slight variations around the $c=7$ value. Due to its rather rigid nature, the 1994 model did not allow the same freedom. However, the key reason for the better performance of the new model is of physical origin: we have abandoned the quasi-normal approximation for the fourth-order moments that we employed in the 1994 model. This means that we have searched for a way to cut down an otherwise unphysical growth of the TOM by

adopting an EDQNM-like procedure. Regrettably, at present we don't have an a priori derivation for (3b,c) which must thus be considered an heuristic suggestion.

V. Conclusions.

The results presented in Fig.1 satisfy the two requirements set out at the beginning, the expressions for the TOMs are simpler than those of the 1994 model and their physical content is better. As a result, the large values of $\overline{w^2\eta}$ and $\overline{w\theta^2}$ that characterized the 1994 model are no longer present and an overall better fit is obtained.

Acknowledgements

The authors are indebted to Dr.D.V.Mironov for kindly providing his LES results which were used in this work.

Figure Caption

Fig.1. The third-order moments Eq.(2) vs. z/h . The LES results of Zilitinkevich et al. (1999) are plotted as dotted lines (the LES data did not contain the $\overline{q^2\theta}$ value); the down-gradient approximation (DGA) model is plotted as dashed lines while the present model results are plotted as solid lines. As well known, the DGA severely underestimates the third-order moments. All TOMs are normalized with Deardorff convective scales $w_*(=2\text{ms}^{-1})$ and $\theta_*(=0.12\text{K})$. The value of the PBL depth h is 1000m.

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